## MIE313

## Heat and Mass Transfer Project

## Group 13

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## Executive Summary

The purpose of this project is to find out witch arrangement of heat sinks will cause the lowest temperature on the board at steady state. The chip arrangement is given in Figure 1 of the lab outline. For our group, the values for a and b in Figure 1 are 40 and 20 millimeters respectively. There are three possible arrangements for the heat sinks: having heat sinks on top of chips 1 and 2 (case 1), heat sinks on top of chips 2 and 3 (case 2), and heat sinks on top of chips 1 and 3 (case 3 ).

In our first lab session we assembled case 1 . We used a power supply that was outputting 23 V and we waited around 30 minutes to reach steady state. The ambient temperature was 22 degrees Celsius. All the measurements are given in the Session 1 Data Record Form. In our second session we assembled cases 2 and 3 . The power supply was outputting 23 V and we also waited around 30 minutes for steady state. The ambient temperature was also 22 degrees Celsius. The measurements for the second session are given in the Session 2 Data Record Form.

We wrote and used the program project.c to find the value for the convection heat transfer coefficient of the aluminum by minimizing the least squares error between measured and calculated temperatures using the Bisection method. We assumed that the heat transfer coefficient of the heat sinks is $50 \mathrm{~W} / \mathrm{m}^{2}{ }^{\circ} \mathrm{C}$. We then used project.c to get the h value for aluminum to get the temperatures around the chips in the cases 2 and 3 . We compared those temperatures to the ones obtained in lab session 2 and they are in good agreement: The average percent error between all the measured and calculated temperatures is only $9.86 \%$.

The measured and calculated values show that case 1 is the one that causes the lowest temperature around the chips.

## Introduction

The purpose of this project is to chose between 3 different arrangements of heat sinks on top of chips: having heat sinks on top of chips 1 and 2 (case 1), heat sinks on top of chips 2 and 3 (case 2), and heat sinks on top of chips 1 and 3 (case 3). The choice is based on which arrangement produces the lowest temperature in the board outside of the chips.

## Experimental Method

In our first session we assembled case 1 . We used a power supply that was outputting 23 V and we waited around 30 minutes to reach steady state. We measured the resistance of the heating pads inside the aluminum blocks (the "chips") and the ambient temperature. The ambient temperature was 22 degrees Celsius. We connected three thermocouples using a bolt and a washer to the top of all the chips and we turned the power supply on. After around 30 minutes we plugged the thermocouples to a digital thermometer to check if the temperatures were not changing by more than 1 degree Celsius in less than 1 minute. This was done to determine approximately if we had reached steady state. All the temperatures passed the criteria. Then we started measuring the boundary temperatures using a thermocouple taped to a thin wooden stick. We measured the temperatures along the boundary every 1 cm (we measured close to one hole, then counted the next 4 holes and measured in hole number 4). The boundary measurements are given in the Session 1 Data Record Form. Then we measured temperatures every 1 cm in between the chips as shown in the Session 1 Data Record Form.

In our second session we assembled cases 2 and 3 . The power supply was outputting 23 V and we also waited around 30 minutes for steady state. The ambient temperature was also 22 degrees Celsius. We checked for steady state and measured in between the chips every 1 cm as described above. The measurements for the second session are given in the Session 2 Data Record Form.

## Numerical Method

The program made to find the h value for aluminum is called project.c and it's found in Appendix 3. The element equation used in this program (using the $\mathrm{m}, \mathrm{n}$ notation found in the textbook) was derived as follows:

(insulation)


Top view

$$
\sum^{4} \dot{Q}_{\text {cond }}+\dot{Q}_{\text {conv }}-\dot{E}_{\text {gen }}=0
$$

Using the convention that heat going out of the chip is positive and going in is negative:

$$
\frac{A}{l} k \sum^{4}\left(T_{m, n}-T_{i}\right)+h l^{2}\left(T_{m, n}-T_{\infty}\right)-\dot{E}_{g e n}=0
$$

Simplifying by substituting $A=l t$ (where $t$ is the thickness of the board and $l$ is the length between nodes) and dividing by $t k$ :

$$
\sum^{4}\left(-T_{i}\right)+4 T_{m, n}+\frac{h l^{2}}{t k}\left(T_{m, n}-T_{\infty}\right)-\dot{E}_{g e n}=0
$$

Isolating $T_{m, n}$ and taking all constants to the other side yields:

$$
\sum^{4}\left(-T_{i}\right)+T_{m, n}\left(\frac{h l^{2}}{t k}+4\right)=\frac{T_{\infty} h l^{2}+\dot{E}_{g e n}}{t k} \quad \text { (equation 4) }
$$

The program uses the boundary conditions stored in the 2D array called temp to write the system equations in the 2D array A and the constants in the 1D array b . It does this by going through each node in the temp array and checking if it is 0,1 or 2 (a node inside the boundary). If it's not 0 it is a boundary node.

If the node has a value of 0 , it writes the second term of equation 4 with the $h$ value of the board in array $A$ and the term on the other side of the equation in array $b$ without the energy generation term. The $h$ value for the heat sink was assumed to be $50 \mathrm{~W} / \mathrm{m}^{2}{ }^{\circ} \mathrm{C}$ and the h value for aluminum starts at $30 \mathrm{~W} / \mathrm{m}^{2}{ }^{\circ} \mathrm{C}$. This value will be optimized using the bisection method.

If the node in the temp array is 1 or 2 , it does the same procedure but it changes the $h$ value for the $h$ value of aluminum and the heat sink respectively and it includes an energy generation term for either the chip or the chip with a heat sink. The energy generation terms were determined by using the following equation:

$$
\dot{E}_{\text {gen }}=\frac{V^{2}}{R N_{\text {chipnodes }}}
$$

Where V is the power source voltage (23V), R is the resistance of the chip without heat sink or the average of the resistances of the chips with heat sinks and $N_{\text {chipnodes }}$ is the number of nodes under the chips. It is assumed all nodes under the chips receive an equal amount of power from the heating pad.

Then the program checks the nodes around the current one to see if they are boundary nodes (with a value higher than 2). If they are, the program subtracts the temperature in the node from the value in $b$ and if not, the program writes a-1 in the corresponding temperature in array A . The program uses the
array temp_num, to know node (and temperature) number based on the node location.

After the system of equations is written in array A, the program uses LU decomposition to solve for the temperatures. The temperatures are written in array x .

The program repeats this whole process changing h using the Bisection method to find the minimum least squares error between the calculated and 3 of the measured temperatures in between the chips. It does this like in the following pseudo code:

$$
\text { h_interval = } 1
$$

while the absolute value of $h$ _interval is higher than 0.001
\{
solve system
calculate least square error
if the least square error is lower than before: add $h$ interval to $h$
otherwise divide h_interval by -10
\}
After minimizing the least squares error, the program finds the maximum temperature on the board outside of the chips and displays it.

The next program in Appendix 3, project2.c, is a program with the same functionality as project.c but without the Bisection method: it uses the $h$ value for aluminum determined in project.c.

## Results and Recommendations

We checked the measured values in our second lab session with the values from project2.c and they agree. The highest percent error between any measured and calculated value is $36 \%$. The average percent error between any measured and calculated value is $10 \%$. For the measured values, calculated values and percent error refer to Appendix 3 and the Session 2 Data Record Form.

The maximum temperatures on the board outside the chips are as follows:

| Case number | Temperature in <br> degrees Celsius |
| :--- | :--- |
| 1 | 92.56 |
| 2 | 95.88 |
| 3 | 92.63 |

Therefore the case with the lowest temperature on the board outside of the chips is case 1.

The simulations and measurements clearly indicate that using a heat sink decreases the temperature of the chips and as a result the maximum
temperature on the board around them. Another conclusion from the area graphs in Appendix 1 is that the temperature of the chips increases when they are closer together so they should be kept as far as possible. Doing this might conflict with other design specifications like keeping the connections between the chips as close as possible to make the processing speed of the system as high as possible.

The main source of error in this experiment comes from the measurement of the spacing between the holes (grid points) on the board and the temperature measurements. Since the distance between the holes was in the order of millimeters the measurement error was high. There was a source of error in the temperature measurements because of the thermocouple itself and the fact that the system reaches steady state in a very long time but we took the measurements only 30 minutes later. Also the insulation below the copper board is not a perfect insulator. Another source of error is the power supply since as the thermocouple heats up, the electrical resistance changes, changing the power output of the resistance heater. There was also error caused by assuming that all the nodes under the chips receive equal amounts of power from the heating pads.

Appendix 1: area and 3D graphs for all cases

Temperature distribution for Case 1: area graph


Distance in $\mathbf{x}$ axis in meters
$\square$ 20-30 $\square$ 30-40 $\square 40-50 \square 50-60 \square 60-70 \square 70-80 \square$ 80-90 $\square$ 90-100 $\square$ 100-110 $\square$ 110-120 $\square 120-130$

Temperature distribution for Case 1: 3D graph


## Temperature distribution for Case 2: area graph



20-30 ■ 30-40 ㅁ 40-50 ㅁ 50-60 ם 60-70 ㅁ 70-80 ㅁ 80-90 ㅁ 90-100 ㅁ 100-110 ㅁ 110-120 ■ 120-130

Temperature Distribution for Case 2: 3D graph


## Temperature distribution for Case 3: area graph




Temperature Distribution for Case 3: 3D graph


Distance in y axis in meters

## Appendix 2: percent error deviation calculations from measured values

The equation used was:

$$
\text { \%error }=\frac{\mid \text { calculated }- \text { measured } \mid}{\text { measured }} \times 100 \%
$$

| For Case 2 <br> Measured <br> temperaturescalculated <br> temparetures |  |  |
| :--- | ---: | ---: |
| Correspond |  |  |
| 79 | 96 | 21.51899 |
| 78 | 94 | 20.51282 |
| 70 | 82 | 17.14286 |
| 68 | 77 | 13.23529 |
| 67 | 81 | 20.89552 |
| 73 | 94 | 28.76712 |
| 80 | 86 | 7.5 |
| 81 | 88 | 8.641975 |
| 77 | 85 | 10.38961 |
| 74 | 81 | 9.459459 |
| 70 | 82 | 17.14286 |
| 67 | 91 | 35.8209 |
| 91 | 89 | 2.197802 |
| 96 | 96 | 0 |
| 97 | 96 | 1.030928 |
| 89 | 90 | 1.123596 |
| 78 | 82 | 5.128205 |
| 70 | 78 | 11.42857 |
| 94 | 86 | 8.510638 |
| 69 | 72 | 4.347826 |
| 98 | 86 | 12.2449 |
| 74 | 68 | 8.108108 |
| 94 | 82 | 12.76596 |
| 69 | 63 | 8.695652 |


| For Case 3 <br> Measured <br> Temperaturescalculated <br> temperatures |  |  |
| :--- | ---: | ---: |
| Cerror |  |  |
| 93 | 85 | 8.602151 |
| 83 | 73 | 12.04819 |
| 80 | 72 | 10 |
| 87 | 84 | 3.448276 |
| 90 | 90 | 0 |
| 88 | 79 | 10.22727 |
| 84 | 79 | 5.952381 |
| 87 | 93 | 6.896552 |
| 95 | 118 | 24.21053 |
| 90 | 112 | 24.44444 |
| 83 | 89 | 7.228916 |
| 79 | 80 | 1.265823 |
| 78 | 81 | 3.846154 |
| 84 | 94 | 11.90476 |
| 85 | 93 | 9.411765 |
| 91 | 92 | 1.098901 |
| 90 | 84 | 6.666667 |
| 87 | 79 | 9.195402 |
| 81 | 79 | 2.469136 |
| 80 | 89 | 11.25 |
| 75 | 84 | 12 |
| 92 | 87 | 5.434783 |
| 93 | 85 | 8.602151 |
| 81 | 80 | 1.234568 |
| 77 | 75 | 2.597403 |
| 78 | 73 | 6.410256 |

Average value for the \%error: $10 \%$.

Appendix 3: programs

